2022 James S. Rickards Fall Invitational

(E) NOTA

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

- 1. The radical $\sqrt{7 + \sqrt{48}}$ can be simplified into the form $\sqrt{a} + \sqrt{b}$. Find 2a + 3b if a < b and a, b are positive integers. (A) 11 (B) 12 (C) 16 (D) 18 (E) NOTA
- 2. Suhas loves to play Valorant. One unfortunate day, he forgot the last 3 digits to his password. Knowing this day would come, Suhas asked Sagar to create the equation below so he can solve for the last three digits a, b, and c. However, Suhas' math skills are rusty from playing Valorant all day so he asks you for help. What are the last three digits of his password assuming that a is the first digit, b is the second digit, and c is the third digit?

- 3. Karthik has been binge watching every episode of MasterChef. Because of this, he has suddenly discovered a newfound passion for cooking. After watching an episode about desserts, he decided he wanted to make some banana pudding. When at the grocery store to buy a banana, he noticed one that had a perfectly parabolic curve with equation $y = \frac{1}{3}x^2$. Find the length of the latus rectum of this curve.
 - (A) $\frac{1}{3}$ (B) 12 (C) 3 (D) 2 (E) NOTA
- 4. Rationalize the fraction $\frac{1}{\sqrt{2} + \sqrt{6} + \sqrt{8}}$.

(A)
$$\frac{3\sqrt{2} - \sqrt{6}}{12}$$
 (B) $\frac{3\sqrt{2} + 8}{2}$ (C) $\frac{\sqrt{3} - \sqrt{2} + \sqrt{15}}{5}$ (D) $\frac{8 + 2\sqrt{2} + \sqrt{14}}{5}$ (E) NOTA

5. Solve the equation for x where $(\log_4 64)(\log_8 12) = \log_4 x$.

- 6. Find the sum of all values of x where |x + 2| + |2x + 10| + |2x 3| |x + 8| = 12. (A) $\frac{-7}{4}$ (B) $\frac{2}{3}$ (C) $\frac{7}{4}$ (D) $\frac{-2}{3}$ (E) NOTA
- 7. Sagar, Sukeerth, and Anish are in the middle of a debate about the eccentricity of a parabola. Sagar says the eccentricity of a parabola is 0, Sukeerth says the eccentricity is 1, and Anish says its eccentricity is 2. Who is correct?
 - (B) Sukeerth (C) Anish (D) All of them (A) Sagar (E) NOTA
- 8. Vibav is currently riding his bike on the line 2x + 3y = 3. He decides he wants Chickfila, which is located on the point (3, 4). What is the shortest possible distance between him and the Chickfila?
 - (A) 3 (B) 2(C) 6 (D) 4 (E) NOTA
- 9. Evaluate the expression $\sqrt{100 \cdot 101 \cdot 102 \cdot 103 + 1}$. (C) 10601 (D) 10301 (A) 10201 (B) 10501
- 10. Natani loves to graph. One day, he decides to graph the polynomial $y = 1281x^8 + 1840x^7 + 1082x^6 + 18$. After this, on the same graph, he graphs the line 18x + 4y = 16. Find the slope of the line passing through the points of intersection of the polynomial and the line.
 - (A) 3 (B) 2(C) 6 (D) 4 (E) NOTA
- 11. Find the number of even sized subsets in a set with 8 elements (the empty set has 0 elements).
 - (A) 2(B) 16 (C) 64 (D) 128 (E) NOTA

2022 James S. Rickards Fall Invitational

12.	Haasini found a new song that she likes to listen to. On the day she discovered the song, she listened to it for 40 minutes. However, because she has a lot of school work, on the next day she can only listen to the song for half as long as the day before. Because of the increasing amount of schoolwork, this pattern repeats forever. After an infinite length of days, how long will Haasini have listened to the song in minutes?						
	(A) 40	(B) 80	(C) 120	(D) 160	(E) NOTA		
13.	Evaluate the sum $\frac{1}{2} + \frac{2}{4} + \frac{3}{8} +$						
	(A) 1	(B) 2	(C) 3	(D) 4	(E) NOTA		
14.	Find the number of terms in the expansion of $(3a + 2b + c + 4d + 5e)^5$.						
	(A) 13	(B) 76	(C) 183	(D) 126	(E) NOTA		
15.	Find the number of distinct permutations of the word ALGEBRA in which the two A's are adjacent.						
	(A) 120	(B) 180	(C) 480	(D) 720	(E) NOTA		
16.	Given that $\log 2 \approx 0.301$, find the number of digits in 4^{16} .						
	(A) 8	(B) 9	(C) 10	(D) 11	(E) NOTA		
17.	Eric is on a road trip to Texas. While on this road trip, he travels through a tunnel that is 18 meters wide and 27 meters tall (assume the arch of the tunnel is modeled by a parabola). What is the height of the tunnel 3 meters away from its center?						
	(A) 24	(B) 22.5	(C) 21	(D) 20	(E) NOTA		
18.	8. Find the value of x if $ x < 1$ and $1 + 2x + 4x^2 + 8x^3 + 16x^4 + \dots = 32$.						
	(A) $\frac{1}{2}$	(B) $\frac{15}{32}$	(C) $\frac{1}{31}$	(D) $\frac{31}{64}$	(E) NOTA		
19.	Kevin loves everything that involves matrices. In fact, he he is known as the Matrix Master. However, Sagar who recently watched Matrix: The Ressurection thinks he is the Matrix Master. Due to this, Kevin asks Sagar a matrix question to test if he is a true matrix master. Kevin asks Sagar to find the element in the second row and third column of the inverse of the matrix below:						

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 6 & 1 & 2 \end{bmatrix}$$

Help Sagar prove he is a matrix master and find the element in the second row and third column of the inverse of the matrix above.

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{-1}{3}$ (D) $\frac{-1}{2}$ (E) NOTA

20. What is the length of the transverse axis of $3x^2 - 12x - y^2 + 6y = 12$?

(A)
$$\sqrt{5}$$
 (B) $2\sqrt{5}$ (C) $\sqrt{15}$ (D) $2\sqrt{15}$ (E) NOTA

- 21. A polynomial with integer coefficients in the form $x^4 + bx^3 + cx^2 + dx + e$ has roots 1 i and $2 + \sqrt{3}$. Find the value of c + d.
 - (A) 1 (B) 4 (C) -6 (D) 12 (E) NOTA
- 22. Sagar is a big Pokemon fanatic. After playing the newest Pokemon game, he decides to embark on his own Pokemon adventure. On his adventure, he finds and catches a Charizard. While flying through the sky he finds two Dragonites 100 miles away flying towards each other. Starting from one of the Dragonites, he zigzags from one to the other until the Dragonites are together. Assuming the Dragonites fly 10 miles per hour and the Charizard flies 30 miles per hour, how far does the Charizard fly in miles until the Dragonites are together (consider the Pokemons' volumes to be negligible in the context of this problem)?
 - (A) 300 (B) 150 (C) 100 (D) 70 (E) NOTA

23. Given that
$$x + \frac{1}{x} = 4$$
, find the value of $x^3 + \frac{1}{x^3}$.
(A) 52 (B) 53 (C) 63 (D) 64 (E) NOTA

2022 James S. Rickards Fall Invitational

24. 20 members of Rickards MAO went to Ms. Cross' room for a club meeting. They all sat around a circular table, and they each flipped a fair coin. Let p be the probability that no two people flipped the same result. Compute $|\log_2 p|$.

$$(A) 20 (B) 19 (C) 18 (D) 17 (E) NOTA$$

25. What is the coefficient of the $x^2y^2z^3$ in the expansion of $(3x + 3y + 4z)^7$? (A) 7500 (B) 7560 (C) 8500 (C)

- (A) 7500 (B) 7560 (C) 8500 (D) 8560 (E) NOTA 26. Evaluate $\sum_{x=3}^{\infty} \frac{1}{x^2 - x - 2}$. (A) $\frac{11}{18}$ (B) $\frac{7}{15}$ (C) $\frac{1}{2}$ (D) $\frac{9}{19}$ (E) NOTA
- 27. Akhil loves to solve systems of equations. One day, he was using Cramer's rule to solve a system of equations with variables x and y. He found that $y = \begin{bmatrix} 3 & 2 \\ 2 & 7 \end{bmatrix}$ Find the value of x + y.
 - variables x and y. He found that $y = \frac{\begin{bmatrix} 0 & 2 \\ 2 & 7 \end{bmatrix}}{\begin{bmatrix} 3 & 6 \\ 2 & 5 \end{bmatrix}}$ Find the value of x + y. (A) 12 (B) 5 (C) -5 (D) 12 (E) NOTA
- 28. A difficult matrix problem has spread fear through the Rickards Mu Alpha Theta Club. Kevin, being the designated Matrix Master, decides to solve the problem on behalf of the club. Help Kevin out and find the determinant of

		$\begin{bmatrix} 19 & 0 & 0 \\ 28 & 16 & 0 \\ 36 & 72 & 8 \end{bmatrix}$		
(A) 0	(B) 1	(C) -1200	(D) 2400	(E) NOTA

29. Viswa loves to watch basketball. His favorite basketball team is the Philadelphia Sixers. Given that the Sixers have a 50% chance of winning any match. What is the probability that they win exactly 3 of their next four games?

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$ (E) NOTA
- 30. Congratulations, you made it to the end of the test! The answer to this question is 12 in numerical form as a single term.
 - (A) 12 (B) Twelve (C) XII (D) 13 1 (E) NOTA